A System-of-System Design of a Guided Projectile Mortar Defense System

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A System-of-Systems Design of a Guided Projectile Mortar Defense System

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A System-of-Systems design methodology is used to evaluate tradeoffs in the design of a guided bullet system for mortar defense. Guided bullets were designed to match the calibers of four different existing auto guns and were modeled in a six degree of freedom simulation. A bullet guidance system was developed based on proportional navigation and several control actuation schemes were modeled. The system simulation was setup to perform Monte Carlo analyses with noise models for various subsystems such as the gun controller and radar. Ranges of gun accuracies and ranges of radar noise were used to create a design space which also included the variation in gun caliber. A design of experiments approach was used to determine the simulation cases that needed to be run to map out the design space. Based on more than half a million independent simulations, a metamodel of the design space was created to capture the interactions between the gun, the projectiles, and the radar. This metamodel allows the user to rapidly evaluate the impact of design tradeoffs and to determine the best system based on his chosen metrics. Available metrics include, cost, weight, defended area, and combinations of the three. Initial results indicate that feasible designs for a guided bullet system are possible within the design space.

I. Introduction

The nature of warfare for the United States has changed considerably in recent years. A traditional large-scale, symmetric conflict is a prospect that the U.S. is unlikely to face in the foreseeable future. Those enemies who might otherwise consider open war on America are restrained by the high probability of defeat associated with facing what is arguably the most advanced military in the world. Opponents have thus turned toward more asymmetric tactics. Relying on patience, anonymity, and the contrivance of fear, recent enemies of the United States hope to disrupt and demoralize while avoiding direct confrontation with the military.

Mortars are easily transported, simple to operate, and indirect fire weapons, which enable deployment from areas which are hidden and outside the line of sight of intended targets. Their projectiles are “dumb” in the sense that once launched they follow a ballistic trajectory, making them immune to counter-guidance techniques. Fortunately, these characteristics also reduce the accuracy of these weapons. Currently, the best way to defend against mortars is to identify and destroy the source, which can prove daunting given the effective range, compactness, and mobility of mortar launchers. Another approach is to destroy the mortar in flight, consequently, mid-air interceptors are receiving significant consideration. The use of unguided interceptors can be effective against mortars though at limited range; the purpose of guiding the interceptors is to extend this range and thus expand the defended area of a single gun.

Given these circumstances, it is the goal of this program to analyze the possible effectiveness, feasibility, and cost of using guided supersonic munitions, maneuvered by pin based actuators developed at the Georgia Tech

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Research Institute (GTRI) and the Army Research Lab (ARL), to intercept and destroy mortar threats mid-air. This paper focuses on the simulation effort and System-of-Systems design methodology used to evaluate tradeoffs in the system design. Results from Monte-Carlo simulations are presented to support the feasibility of the concept.

II. Background

A system-of-systems is a collection of systems that have operational and managerial independence from one another, but collaborate together to achieve a common purpose[1]. A system-of-systems approach can be used to describe events at every level by functionally decomposing, or structurally breaking down, the components of the entire system into a hierarchy of decision making levels. For an in flight mortar interception, there are three independent systems involved: the gun system, the radar system, and the projectile (interceptor) system. For proper system engineering, the designer must overview the entire system (comprised of its components), and have an understanding of how those components interact.

A Monte Carlo simulation is a way to generate information when events occur in a random way, using random sampling in a computer simulation in which the results are generated repeatedly to develop statistically reliable answers. Uncertainty in a system can be quantified by adding distributions to any point values, such as any inputs designed as part of the design's degrees of freedom. Many cases with randomly selected points from these distributions can then give a statistical indication of the probability with which each response will occur. A Monte Carlo simulation can also be used to conduct a design space exploration. This involves populating the design space with many combinations of randomly selected values for all of the design variables, and simultaneously viewing the response space. When designing to a specific goal in an extremely multimodal and/or discontinuous design space, this method may prove to be more efficient than making use of an optimizer that relies on smooth continuous functions and often only find local optima.

To be able to conduct an uncertainty quantification analysis or design space exploration, a modeling and simulation tool is required, and must be executed many times to introduce sufficient variation. Initially, this project used commercially available software to perform these tasks. While available software is suitable for determining the characteristics of a single projectile, no commercial solutions were found capable of multiple projectile simulations, which are required to model a bullet-mortar intercept. Consequently, significant effort was devoted to developing object-oriented code that would simulate the dynamic behavior of multiple projectiles simultaneously. This code is called the Extended Area Protection System (EAPS) and its core six DOF model was adapted from BOOM, an exterior ballistics simulation program[2]. EAPS reads in the aerodynamic and mass properties of each projectile directly from a file generated by the modeling software. This allows users to design a projectile using third party software, then import it into EAPS for multi-body simulation. Other modules are included in EAPS including models of radar, a guidance system, projectile control mechanisms, various sensors, and signal filters. Because of its object-oriented design, the software can readily model the trajectories of multiple mortars and interceptors as well as individual sensors and guidance systems for each round.

The EAPS simulation environment is fairly complex, executing a significant number of computations that often require several seconds for a single trial run, even when running as native binary code on a high speed computer. For this reason, a surrogate model was used to reduce the computing hours needed to a reasonable time period. A surrogate model (or also commonly referred to as a metamodel) represents a small segment of a more complex analysis tool, usually in the form of an analytic equation based on a regression of statistical inputs and response metrics[3]. This model allows for a rapid calculation of responses by employing simple equations to relate independent variables to responses. A neural network based response surface equation modeling approach was used to create surrogate models that are capable of handling multimodal and discontinuous design space of this problem.

For a system-of-systems environment comprised of a Monte Carlo based design space exploration employing rapid surrogate models, both bottom-up and top-down design analysis may be executed simultaneously[4,5]. This process enables any response to be treated as an independent variable, meaning that information can flow in either direction within the hierarchy. For bottom-up design, the effect of manipulating component design variables on system performance (or other metrics of interest such as cost), as well as the effect of the system performance on higher level metrics within an operational environment can be evaluated. For top-down design, placing limits or constraints on high-level metrics will drive the required component performance. When the design space is completely mapped out, any design region may be visualized using a collection of multivariate plots.

The top-down approach will likely define several combinations of design variables that meet requirements at various levels in the hierarchy. At this point, current system and subsystem designs may be evaluated within the

architecture. However, bottom level design variables drive system effectiveness, but top level requirements may require performance beyond what is available. Therefore this paper utilizes a methodology that enables decision makers anywhere across a system-of-systems hierarchy to rapidly and simultaneously manipulate the design space, however complex. This process eliminates the iterative steps that usually occur when dealing with flowing requirements from one level to next lower in the systems engineering process. By removing the internal constraints local to one level in the systems hierarchy, the design space could be manipulated from the top-down of a design environment directly, without having to deal with the iterations involved in optimizing a system or subsystem one level at a time.

III. System Descriptions

A. Gun System

The gun system ultimately envisioned by this project is one that is aimed by a fire control computer based on information from the radar system. This fire control computer would estimate the trajectories of the target and the estimated time to intercept, and aim the gun and fire a projectile. This process would be repeated for multiple projectiles based on the firing rate of the gun, the type of threat and the type of projectile being fired. Practically, it is very difficult to hit such small targets at long ranges due to various sources of error. These errors can be categorized as radar errors and gun errors at least for unguided rounds. The radar errors will be described in greater detail below, but they do affect the initial pointing error of the gun due to the inexact location of inbound threat. The gun system errors include the pointing error introduced by the radar and an additional pointing error due to mechanical differences in the actual gun position and the commanded gun position. Errors are also introduced by tip off errors and by cartridge to cartridge variations in both the charge and the actual projectile. All of these errors stack up, but to reduce the number of variables simulated these errors were rolled up into errors in gun elevation and azimuth, and muzzle velocity. As an example, the errors used for the first simulation are shown in Table I.

Table I. One-Sigma error, for gun pointing/firing.

	Pointing Elevation (mrad)	Pointing Azimuth (mrad)	Muzzle Velocity (m/s)
min	0	0	0
max	1.25	1.25	2.50

B. Radar System

The radar system is assumed to be earth-fixed and conceptually located at the same place as the gun system. As all radar systems do, the system modeled in EAPS introduces error. Error is simply defined as the difference between the measured location of the object of interest, and the true physical location of that object in three dimensional space. Radar produces errors of two types: bias and noise. A bias occurs when the radar reports values that are off by a constant amount and in a constant direction after each sweep. For example, the radar may report a target to be 1m east of its true location every cycle. Noise, on the other hand, occurs in different magnitudes and different directions each sweep. One cycle it may report the projectile to be 1 cm north of its true location, and the next it may report it as half a meter southwest of its true location. Although EAPS is capable of simulating both radar noise and bias, bias was set to zero in all cases. This decision was made under the assumption that a constant radar bias could be detected and accounted for by the fire control computer on the ground. Random noise, on the other hand, can not be calibrated out of the system and must be overcome by course corrections induced on the projectile system. Errors simulated in the radar system included sensing ability of elevation, azimuth, range, and roll orientation.

1. Radar Error Estimation

Statistical methods can be used to describe the error of a radar measurement. The most accurate method of error measurement is with using actual test data; however this is not very practical when required for use in a simulation with changing properties. Mathematical approximations can be created that model actual error components and enable accurate analysis of a particular system's error. In most cases, a normal distribution may approximate radar noise accurately. Of course, the actual shape of the error distribution may not be normal, but Barton[6] states that "the

normal distribution is often assumed to represent errors of unknown characteristics, and it closely approximates the distribution of many actual errors".

Using a relation for the phase difference between the received signals between two interferometer antennas[7], Holder[8] derives Equation 2 relating the measurement error standard deviation for an interferometer radar, as a function of radar properties.

$$\sigma_\theta = \frac{\lambda}{2\pi D \cos\theta \sqrt{SNR}} \tag{2}$$

If the radar is assumed to be pointing directly at the target, the offset angle θ goes to 0, and the cosθ term goes to 1, therefore relating radar measurement error standard deviation to wavelength (λ), the distance between interferometer antennas (D, a measure of radar size), and signal-to-noise ratio (SNR). The SNR can be found using Equation 3, using pertinent radar subsystem level properties[6], including transmitted power (P_{avg}), transmit antenna gain (G_t), receive antenna gain (G_r), wavelength (λ), radar cross section (RCS), observation time (t_0), transmitter/receiver range to target (R), Boltzmann's constant (k), and system input noise temperature (T). A representative range for the radar subsystem model is provided in Table II.

$$SNR = \frac{P_{avg} G_t G_r \lambda^2 (RCS) t_0}{(4\pi)^3 R^4 kT} \tag{3}$$

Table II. Variables and Bounds Used for Radar Subsystem Model.

Property	Minimum	Maximum	units
Power (average)	100	250	W
Gain (transmit)	12	50	dB
Gain (receive)	15	40	dB
Frequency	14	18	GHz
Range	500	2000	m
RCS	-25	-15	dBsm
Dwell Time	0.001	0.005	sec
Distance between Antennas	0.5	10.0	m

2. Filters

In these type of engagement scenarios, Kalman filters or other techniques are used to smooth the radar noise and provide cleaner tracks. For the present simulations, a much simpler type of filter was used which basically reduced the simulated noise as the number of samples increased. It was assumed that the noise associated with the position of the target or outbound projectile could be reduced by

$$\frac{3\sigma}{\sqrt{N}} \tag{4}$$

while the noise associated with the velocity could be reduced by

$$\frac{192\sigma}{T\sqrt{N}}. \tag{5}$$

where N is the number of measurements by the radar. For the mortar, N was set to 81 at the beginning of the simulation and allowed to increase as it was assumed that the radar had been tracking the mortar for some time prior to interceptor launch. For the interceptor, N was set to 9 at launch as it was assumed that some information about the projectile's location could be assumed even though there were no prior returns on which to base the radar's estimation of the projectiles position and velocity.

3. Radar Cost

Holder[8] gives a radar cost estimation based on the number of antenna elements. This basic relation gives radar cost as the cost per radar antenna element times the number of elements, multiplied by a factor of 4. A reasonable estimation for the cost of an individual element is $1000. The total amount of elements in one radar antenna array can be estimated as the square of the number of elements along one side of a square shaped antenna array. The number of elements on one side of the array is just the length of one side of the antenna array divided by the space between the elements in the array, plus one. The length of one side of the antenna array is simply the square root of the calculated antenna area (A), and the space between elements can be approximated by one half the wave length of the radar. Noting that it takes two antenna arrays for an interferometer to measure angle in one plane (i.e. either elevation or azimuth), it takes at least three to measure both elevation and azimuth. Therefore, the number of radar elements can be found using Equation 6,

$$Number\ of\ Radar\ Elements = \left(\frac{\sqrt{A}}{0.5 * \lambda} + 1 \right) \cdot \left(Number\ of\ Antenna\ Arrays \right) \tag{6}$$

where antenna area is found from antenna theory[7], given receive gain, wavelength, and the efficiency of the antenna aperture (η)

$$A = \frac{G_r \lambda^2}{4\pi\eta} \tag{7}$$

C. Projectile System

Nearly three years of experimental tests at the Georgia Tech Research Institute[9,10,11] and the Army Research Lab (ARL)[12,13] have demonstrated that a pin based actuator can develop the forces required to steer a supersonic projectile. For the current study, projectiles with a similar form factor and mass distribution were designed and modeled, shown in Figure 1, using a standard ballistic analysis code,[14] which was used to calculate inertial and aerodynamic properties. These rounds were designed to be KE rounds that were command guided into the target. Notionally the rounds consist of various subsystems which include the penetrator, the guidance actuators, inertial measurement system, and onboard guidance computer. As shown in Figure 1, the penetrator notionally makes up 75% of the projectiles mass and the electronics and actuators makes up 4% of the mass and is moved to set the cg. The remaing 21% of the mass is the structure and fins. The steering forces developed by the guidance pins were modeled based on results from wind tunnel tests at GTRI and live-fire range tests at ARL. The range tests were recently completed at the Army Research Lab's range facilities using ten test articles (see Figure 2) to further refine and understand the effects of pin-forces at supersonic speeds.[15] These range tests served to provide very accurate aerodynamic coefficients for the six degree-of-freedom (6DOF) simulations. Four discrete interceptor projectile sizes were considered based on these previously tested rounds and of sizes to match existing auto guns, i.e., 30mm, 40mm, 50mm, and 57mm. This family of self similar projectiles is shown in Figure 3.

The guidance module in EAPS uses a classical proportional navigation guidance (PNG) scheme to direct the bullet toward an intercept. PNG generates three axis acceleration commands proportional to the angular rate of the line-of-sight between the bullet and mortar. Since the radar is earth-fixed, the acceleration commands are computed in the inertial frame and rotated into the projectile's body frame. The inertial-to-body frame rotation includes the effect of noise on the projectile's attitude estimation. Because the bullet cannot generate longitudinal thrust, only the lateral component of the acceleration command is used by the projectile's controller.

Several controller schemes were examined to determine which one is the most effective in implementing the guidance commands. Initial designs for the pin mechanisms on a non-spinning projectile called for one set of asymmetric pins for roll control and a second, symmetric set for pitch control. Consequently, a control scheme was developed to roll the projectile's pitch control into alignment with the commanded lateral acceleration vector. A classical proportional, integral, derivative (PID) controller was then used to pitch the projectile in order to follow the commanded acceleration.

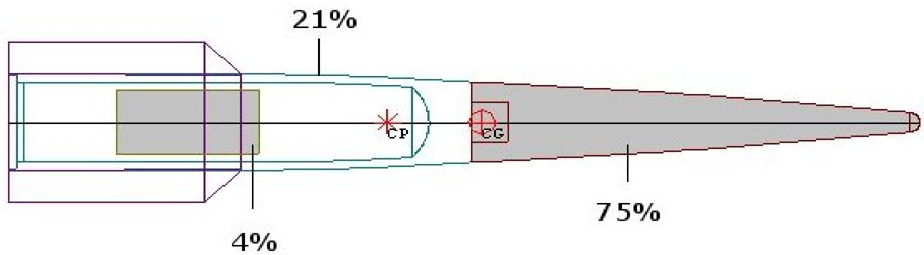

Figure 1. Model of the supersonic, fin stabilized projectile.

Figure 2. Range test projectiles.

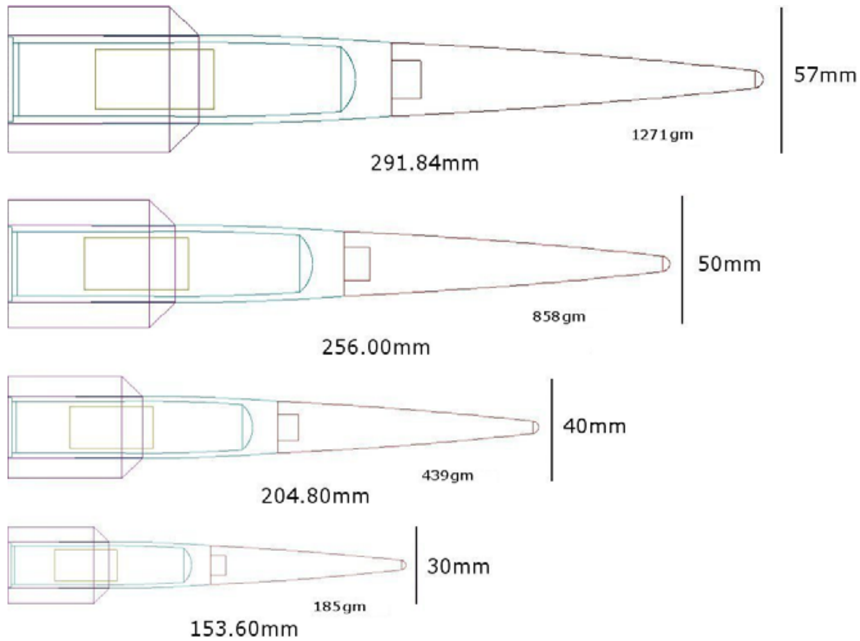

Figure 3. Length, diameter, and mass of the four scaled projectiles.

IV. Initial Simulations and Observations

Before conducting the system of system analysis, it was first necessary to simulate a large number of projectile-mortar engagements. For this study, a single representative mortar trajectory was considered with a desired intercept at an altitude of 75 m. To speed up the 6 DOF simulations, each simulation was initiated with the mortar already in the air and only the last few seconds of its flight were considered.

Monte Carlo simulations were run varying multiple parameters. In the Monte Carlo simulations of each engagement scenario, the target and interceptor physical characteristics remain constant. This assumed that the variations in inertial and aerodynamic characteristics, which might indeed occur in part due to imperfect manufacturing of projectiles, were negligible. Five discrete intercept ranges were considered; 250m, 500m, 1000m, 1500m, and 2000m. Simulating engagements for each bullet size at each range generates 20 unique sets of Monte Carlo simulations each with several variables that must be tested individually. The continuous variables in this first batch of simulations included all three sources of error from the gun system (elevation pointing error, azimuth pointing error, and muzzle velocity variation- Table I) and all four sources of error from the radar system (elevation, azimuth, range, and roll sensing - Table III). Given these seven variables, a seven dimensional design space was created and one-sigma errors were produced as shown in each of the tables.

Table III. One-Sigma errors, centered about mean values, for Radar sensing.

	Elevation (rad)	Azimuth (rad)	Range (m/km)	Projectile Roll Orientation (mrad/m)
min	1.50E-05	1.50E-05	0.05	45
mid	0.0005075	0.0005075	0.525	107.5
max	0.001	0.001	1	170

From these error sets, 178 cases were generated based on a Design of Experiments (DoE) methodology. Each case in the DoE is composed of a set of unique initial values for each of the seven variables being tested. Each set of initial conditions, however, does **not** produce one unique set of end conditions. This is due to the fact that radar noise is random from run to run.

In each set of initial conditions the values for radar error represent the mean of that error. During each radar sweep, the simulated radar reports back a value that is randomly noisy according to a Gaussian distribution centered on this mean. For this reason, the exact same set of gun errors and radar errors may produce a kill in one instance, and a miss in the next, depending on the noise level. To get a good sense of how the interceptor would perform in any given case, it was therefore necessary to perform multiple simulations and extract a probability of hit. Each of the 178 cases was thus repeated 100 times, swelling the number of simulations to 17,800 per discrete case. The choice of 100 Monte Carlo simulations was selected such that the resulting probabilities of hitting the target were accurate within at least 1%. More Monte Carlo simulations per 6-DoF case would have been desired, however, with 20 discrete cases, using 100 Monte Carlo simulations per 6-DoF case presented a total of 356,000 simulations to be carried out in order to fulfill the requirements of the system-of-system analysis scheme.

A collision detector module determines whether the engagement is a HIT or a MISS based on the geometry of the interception. This routine models the bullet and mortar as cylinders of the appropriate length and diameter. The cylinders are oriented according to the bullet/mortar's attitude and positioned according to the c.g. locations in inertial space. A HIT is scored if the cylinders intersect during the simulation.

Upon analyzing the entire set of simulations, several important facts became evident. It quickly became clear that radar noise was having the greatest influence on the success or failure of the controller, especially sensing error of Elevation and Azimuth. Two representative cases are compared at a range of 500 m for both a noisy and a not so noisy radar. A plot of the closest pass of the interceptor to the mortar in 3D space is shown in Figure 4 where it is clear that many of the interceptor rounds miss by over a meter when the radar noise is high. Zooming in on the same data, Figure 5, one can see that when the controller is fed accurate radar data it is able to guide the interceptor into the target. Other observations were also made such as the fact that gun pointing error ceased to have much importance at longer ranges for the guided rounds, but was extremely important at close in ranges as the projectile had little time to correct for these errors.

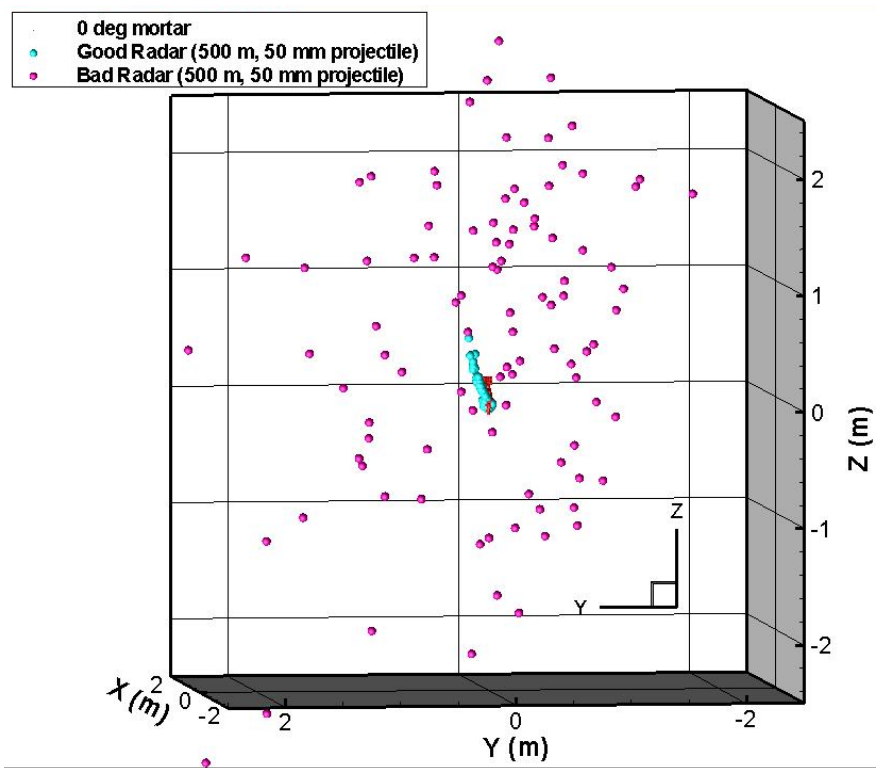

Figure 4. Distribution of controlled 50 mm rounds at 500 m range for low and high radar noise cases.

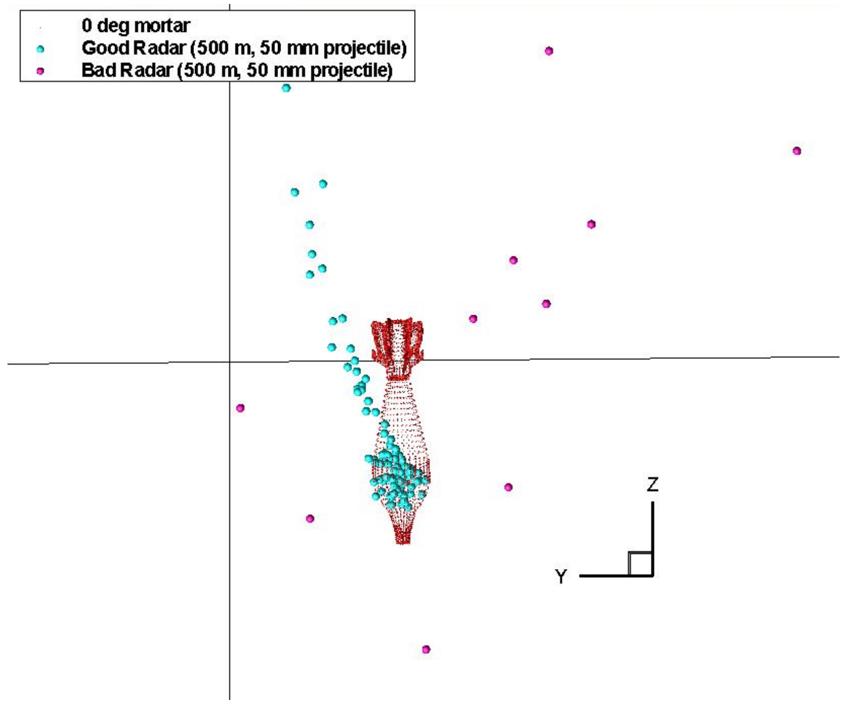

Figure 5. Distribution of controlled 50 mm rounds at 500 m range for low and high radar noise cases.

One way to explore all of the effects of the independent variables upon a desired metric is through the use of multivariate plots. A multivariate plot of the present data set is shown in Figure 6 represents the over 380,000 runs with a large set of possible design points. Each small graph is a trade space between two independent variables and is mirrored across the diagonal axis. A design metric based on a 10% confidence that the round missed the mortar by less than some distance was chosen for the initial evaluation. When this design metric is compared to the intercept range, as highlighted by the green box and enlarged in the inset, it is possible to start down selecting those designs which meet certain criteria. First, only those designs that yielded intercepts that on average had a 10% or greater chance for each round to pass within at least 0.75 m of the mortar were selected, and these points are shown on all the plots by both the green and red points. When the additional criteria is added that the intercept also must occur at a range greater than 1000 m, then the valid designs are indicated only by the green points. By putting these restrictions upon the design space it is thus possible to exam which variables have the most influence upon the desired outcome, for example it is found that there are no 40 mm gun solutions that are effective. Further it is found that the radar azimuth and elevation noise must be low for the selected criteria to be met. As shown in Figure 7, which is an enlarged version of the subplot shown in Figure 6 with the designs that do not meet the criteria hidden, as the criteria becomes more stringent, the system must have a combination of low radar elevation and azimuth noise, but it is possible to trade noise in one axis for the other and still meet the criteria. On the other hand, it is found that the gun errors have almost no effect at these ranges as shown in Figure 8 for the same design criteria. In this plot, it is seen that there are no clear trends indicating that there is little correlation with gun noise and being able to hit the mortar at ranges above 1000 m. Another way to capture these trends, albeit with less detail, is to use a Pareto plot as shown in Figure 9, which shows the relative influence of each of the variables on the projectile miss distance at a range of 2000 m.

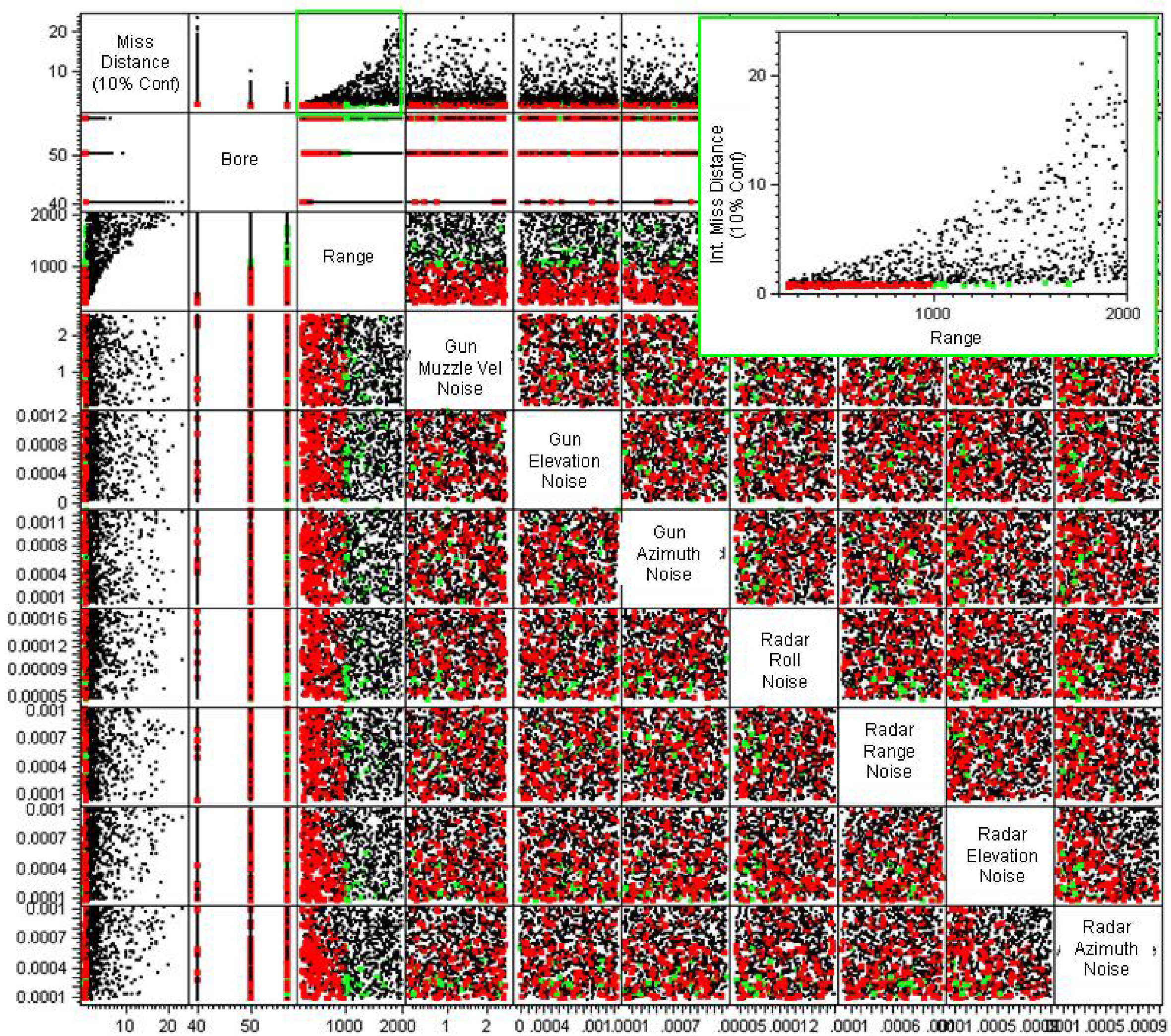

Figure 6. Multivariate plot for 3 projectile, 8 noise sources, trade study.

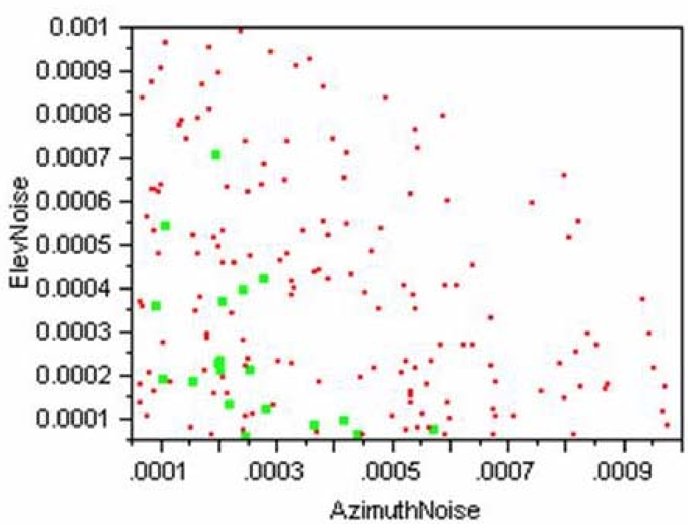

Figure 7. Effect of radar noise upon chosen design space.

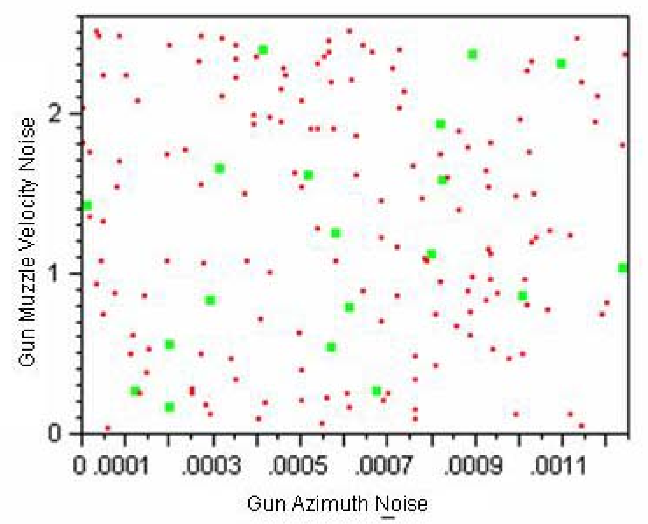

Figure 8. Effect of gun noise upon chosen design space.

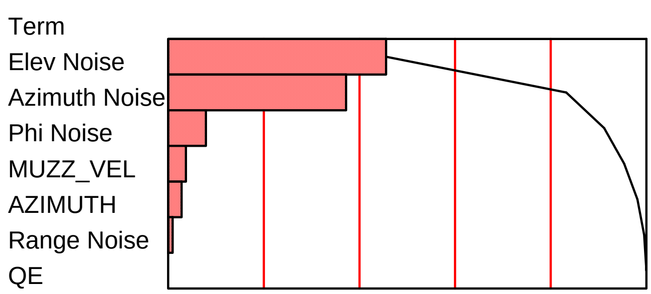

Figure 9. A Pareto Plot, 50mm Bullet at 2000 m.

V. System-of-Systems Simulations

A. Reducing the Design Space

Even though several hundred thousand individual scenarios were completed, distributed across dozens of CPU's, there was not enough data provided to develop the desired metamodels. One solution would have been to increase the number of runs, to perform the estimated extra 1.4 million scenarios necessary. Instead, insights from the initial set of runs were used to more intelligently select the variables of the design space. First, the 30mm and 40mm rounds were thrown out due to their ineffectiveness. The 57mm round was also thrown out, as it was merely a more expensive version of the 50mm in terms of performance. Also, because of the limited effect gun system errors had, these were not considered. Also dropped were the two less significant radar sensing errors, roll orientation and range. In addition to the two remaining variables (azimuth and elevation noise), two more were introduced. In this set of scenarios, the radar update rate and the control update rate also varied.

The bounds for these variables are shown in Table IV. The lower bound of each of the radar noises is intended to represent advanced technology radar, which represent the lowest noise and therefore highest accuracy. The upper bound is intended to represent a low cost, low technology radar, which has the highest amount of noise, and therefore also the least accurate possible. The radar update rate is the rate at which projectile and target position updates are sent to the fire controller, and the guidance update rate is the rate at which guidance updates are sent to the projectile to allow for course correction.

To create the neural network surrogate models, a Design of Experiments (DoE) consisting of 75 runs was created by combining a Central Composite Design (CCD) consisting of 25 runs to capture the corners of the design space, a Latin Hypercube Sample (LHS) consisting of 25 runs to capture multimodal effects within the bounds, and 25 uniformly selected random points to be used as a hold back sample to validate the model fit. Recall that each trial of the DoE executes a Monte Carlo simulation within the 6-DoF code. Because the radar error is represented by random noise, each run of the DoE input settings results in a different set of radar noises, each one requiring a Monte Carlo simulation to sample from the normal distributions describing each noise variable. Multiple runs are required to quantify the confidence of achieving a response; here the responses of interest are the minimum miss distance of the interceptor and target, and probability of direct hit. For this set of simulations, each DoE case was repeated 500 times.

Table IV. Variable Bounds for the second set of 6-DoF Simulations.

Property	Minimum	Maximum	units
Radar Elevation Noise (1σ)	0.025	0.500	mrad
Radar Azimuth Noise (1σ)	0.025	0.500	mrad
Radar Update Rate	10	50	Hz
Control Update Rate	90	200	Hz
Intercept Range	500	2500	m

B. Multiple Projectiles Analysis

Another modification to the prior set of analysis was that the effect of firing multiple interceptors at the target was now considered. To achieve a desired probability of hitting a target, it was found that multiple rounds were needed. Equation 8 was used to determine the probability that a target will be hit at least once by *n* projectiles, given the probability of hitting a target with one projectile is $P_{\text{hit, single shot.}}$ Driels[16] can be consulted for the derivation of a similar relation from the binomial distribution, which is a set of repeated trials of the same experiment in which each of the trials are independent. A major assumption used in this study is that the probability of hitting the target is the same for each projectile (a very common assumption made when dealing with "bursts" of projectiles assumed to hit the target at the same intercept range [17]).

$$P_{hit,\text{multiple shot}} = 1 - \left(1 - P_{hit,\text{single shot}}\right)^n \tag{8}$$

C. Assembled Hierarchical Model

After running the second large set of data for only the 50 mm round, it was possible to create metamodels and then populate the design space with a Monte Carlo approach. A multivariate plot of the populated design space is shown in Figure 10. A Monte Carlo simulation sampling from a uniform distribution selected 1,000 samples from each variable within the bounds was used to create the surrogate models. This includes the intercept ranges, the two radar noises, the two update rates, and the radar subsystem variables that proved to have the most impact on the variability of the responses. For each combination of variable values selected, the responses of interest are tracked. Therefore, each corresponding Monte Carlo simulation has a unique array of variable and response values, and all variables and responses of interest are shown on the multivariate scatter plot, which is essentially a collection of bivariate plots. Because the independent variable values are samples from a uniform distribution, the points evenly fill the design space between them. The dependent responses are calculated using the variables.

The design space shown in Figure 10 is sorted by number of projectiles fired, and those corresponding design points in turn appear in every other response/response, response/variable, and variable/variable bivariate plot. Figure 10 provides an example where a decision maker would state a desired probability of hitting the target of 0.70. Comparing the multiple versus the single shot probabilities of hit, it is clear that a one shot solution does not meet the threshold, and with increased shots fired, the required single shot Phit is reduced. In this example, a limit of $5M cost on the radar could also be set as shown on the insert in the upper right of Figure 10 where all of the design points in the hierarchy that do not meet these high level desires have been hidden.

When the design points that do not meet the requirements are eliminated, several interesting trends emerge in the component level, and can be viewed as extracted bivariate plots from the original multivariate plot. Figure 11 shows the multiple shot Phit versus the radar noise and the distance between the radar antennas. Note the very obvious tradeoff space that emerges in both plots. With increased number of rounds fired, the allowable radar noise decreases, and the required distance between antennas can be decreased. Note that all of these points meet the cost constraint discussed for Figure 10.

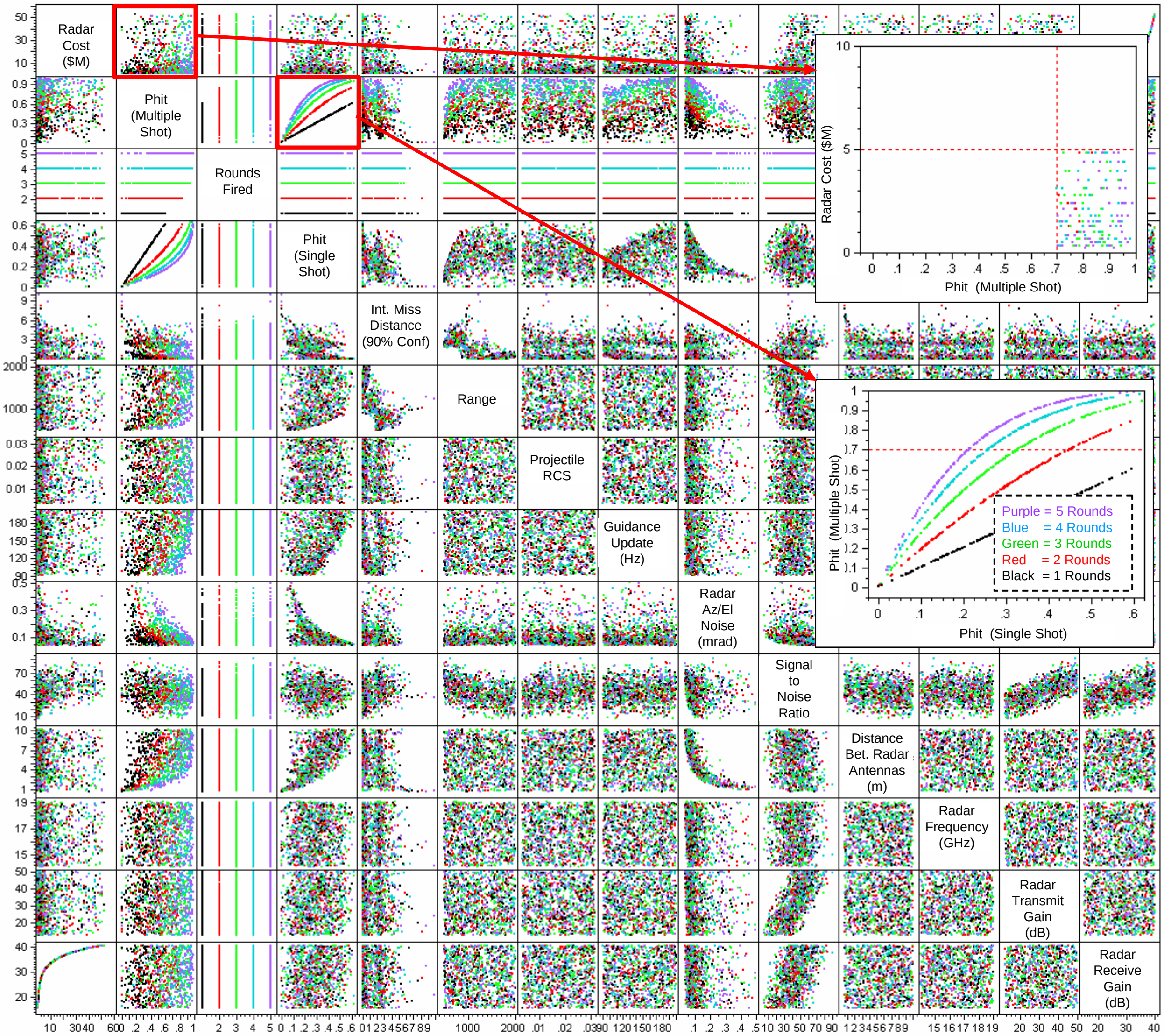

Figure 10. Top-Down Hierarchical Design Space Exploration Using a Multivariate Scatter Plot.

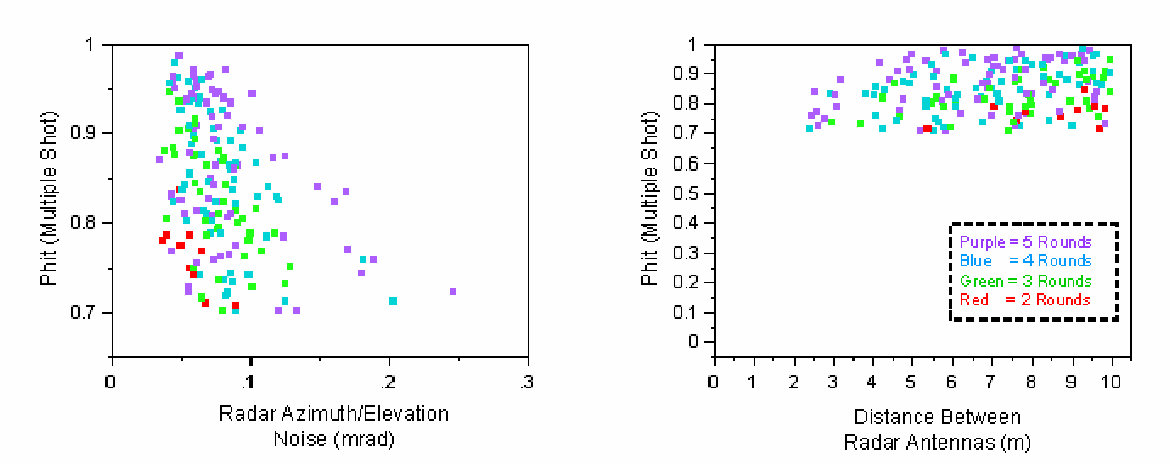

Figure 11. Bivariate Plots of Multiple Shot Phit versus Radar Capability and Size.

Taken further down the component level of the radar properties, Figure 12 shows on the left plot the tradeoff between guidance update rate and distance between radar antennas. Recall that these two properties were independent variables, populated in this design space using a Monte Carlo simulation selecting points from a uniform distribution, therefore this bivariate plot would have originally been uniformly populated with design points. However, as the top level constraints filtered out designs that did not meet the Phit and cost requirements, the remaining design points are more concentrated in the region of high guidance update rate and larger distance between antennas. This is clearly intuitive , however this allows a decision maker to instantly make these kind of tradeoffs, as well as observe how the effects of firing multiple projectiles further constrains the design space. For example, with less rounds fired against the target, the higher the guidance update and larger the distance between antennas must be. The right side of Figure 12 shows the distance between antennas versus the radar frequency. Note again how the design points are more concentrated at larger distance between antennas, however the effects of frequency are more evident with fewer projectiles fired. At lower frequencies and with only two rounds fired (the larger red design points), a very large distance is required between the antennas, however as frequency is increased, the radar size can be reduced. These type of design trades driven by top level requirements, help steer the system design while maintaining the overall system-of-system performance.

While only one example, it is possible to see how the choice of overall system performance metrics can constrain the design space and lead to appropriate system requirements for each of the subsystems. Not shown as an example, but an equally valid approach, is to perform an inverse type of analysis, where one could see how improving technology at the component level, such as increasing the radar gain, could increase overall system performance or reduce technology requirements upon one of the other sub systems.

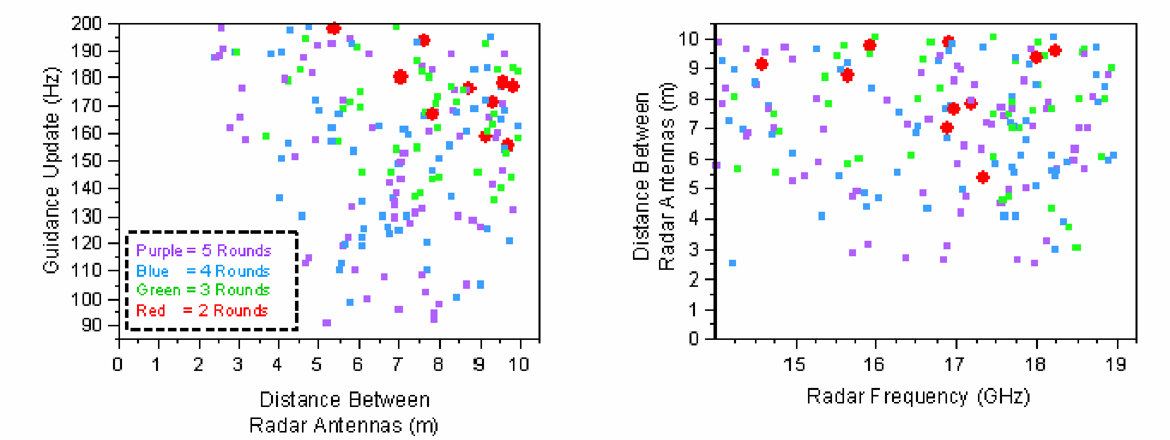

Figure 12. Bivariate Plots of Various Radar Properties Tradeoffs.

VI. Summary of Results

In this paper the development of a system-of-systems architecture for the evaluation of an air defense providing protection against mortars using guided projectiles was discussed. While focusing on this particular system, the methodology is applicable to other problems. The results utilized a methodology that enables decision makers across a system-of-systems hierarchy to rapidly and simultaneously manipulate the design space, however complex. This process eliminates the iterative steps that usually occur when dealing with flowing requirements from one level to next lower in the systems engineering process. By removing the internal constraints local to one level in the systems hierarchy, the design space could be manipulated from the top-down of a design environment directly, without having to deal with the iterations involved in optimizing a system or subsystem one level at a time. Rather than extract single curves to show the pertinent tradeoff space between responses and variables for all other variables fixed, this method keeps all degrees of freedom open throughout the design hierarchy. This allows for constraints to be placed anywhere in the hierarchy, regardless of whether they were variables or responses, clearly identifying the feasible design options and tradeoffs between any combination of variable/variable, variable/response, and even response/response.

For the case under consideration, it was shown using 6 DOF simulation and design space exploration that a guided projectile system could be used for air defense provided that radars of sufficient accuracy were available. A similar type of analysis could be performed for other defensive concepts such as a missile based system. Further development of the system could also evaluate the defensive capabilities against other airborne threats such as rockets or artillery. Finally, the methodology described has utility in evaluating any large system that consists of multiple subsystems that need to interact to achieve a larger goal.

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